

COMPARATIVE STUDY OF OPTIMALITY IN ASSIGNMENT PROBLEM: PROPOSED METHOD AND EXISTING METHODS

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Abstract: *The assignment problem is a special case of the transportation problem in which the objective is to assign a number of resources to the equal number of activities at a minimum cost (or maximum profit). It is one of the main problems while assigning task to the worker. It is an important problem in mathematics and is also discuss in real physical world. In this paper we present a new proposed method for getting optimality of assignment problem. The results of new approach are compared graphically with existing approaches namely Hungarian method and Matrix Ones Assignment method (MOA method), and this approach outperforms other methods and also show the efficiency of the proposed methodology by giving some numerical examples.*

Keywords: *Assignment problem, Hungarian Assignment method (HA-method), Matrix One's Assignment method (MOA-method), Proposed method, Optimization.*

Introduction

Assignment Problem is one of the fundamental combinatorial optimization problems in the field of Operation Research in Mathematics. It is one of the special cases of transportation problems. An ideal way to put straight forward immediately after the transportation problem is the assignment problem. The assignment problem comes under the type of linear programming problem in which our main focus is to assign n number of tasks to m number of workers at a minimum cost/maximum profit. Assignment problem may be any types of problem for assigning like person to jobs, teacher to classroom, operators to the machine, driver to bus, bus to delivery routes etc. Balanced assignment problem is a problem where $n = m$. Otherwise the problem is said to be unbalanced assignment problem if $n \neq m$. This is particularly important in the theory of decision making. This problem finds numerous applications in production planning, telecommunication, VLSI design, economics etc. In a normal case of assignment problem where

the objective is to assign the available resources to the activity going on so as to get the minimum cost or maximum total benefits of allocation. In this paper we developed an efficient method for assignment problem as compared to the existing methods. The corresponding method has been formulated and some numerical examples have been considered to illustrate the method. Finally we compare the optimal solutions among new method and two existing methods.

The Assignment Problem introduced by **Votaw and Orden** can be solved using the linear programming technique. Perhaps the best known, most widely used method for solving the standard assignment problem is the "Hungarian Method" originally developed by **Kuhn** in 1955. **Balinski and Gomory** introduced a labeling algorithm for solving the transportation and assignment problems. **Ford and Fulkerson** introduced the models and algorithms in Flows in Networks which are used widely today in the fields of transportation systems, manufacturing, inventory planning, image processing, and Internet traffic. **Aggarwal** developed two algorithms for solving bottleneck assignment problems. **Kadhirvel and Balamurugan** solved the unbalanced assignment problems using triangular fuzzy Numbers. There are various optimization methods to solve the assignment problem like Matrix Ones Assignment, Genetic algorithm, Lexi-search approach, Primal method etc. Different methods have been presented for Assignment Problem and various articles have been published on the subject for the history of these methods.

Different methods for solving Assignment Problem:

- (i) Complete Enumeration Method
- (ii) Transportation Method
- (iii) Simplex Method
- (iv) Hungarian Assignment Method

Mathematical Formulation of Assignment Problem

Each assignment problem has a matrix associated with it. Generally the row contains the objects or people we wish to assign, and the column comprises the jobs or tasks we want them to assign. Consider a problem of assignment of n resources (workers) to n activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job.

The cost matrix is same as that of a transportation problem except that availability at each of the resource and the requirement at each of the destinations is unity (due to the fact that assignments are made on a one-to-one basis).

Let x_{ij} denote the assignment of i^{th} resource to j^{th} activity, such that

$$x_{ij} = \begin{cases} 1; & \text{if resource } i \text{ is assigned to activity } j \\ 0; & \text{otherwise} \end{cases}$$

Then, the mathematical formulation of the assignment problem can be stated as follows:

$$\begin{aligned} &\text{Minimize the total cost } z = \sum_{i,j=1}^n c_{ij}x_{ij}, i = 1, 2, \dots, n; j = 1, 2, \dots, n \\ &\text{cost } z = \sum_{i,j=1}^n c_{ij}x_{ij} \end{aligned}$$

Subject to the constraints:

$$\sum_{j=1}^n x_{ij} = 1 \text{ and } \sum_{i=1}^n x_{ij} = 1; x_{ij} = 0 \text{ or } 1$$

for all $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, n$.

In the above statement c_{ij} is the cost associated with assigning i^{th} resource to j^{th} activity and x_{ij} denotes that j^{th} job is to be assigned to the i^{th} person.

In this paper we developed an efficient method for assignment problem as compared to the existing methods. Some numerical examples are given to demonstrate the procedure of the proposed method. Finally we compare the optimal solutions among new method and two existing methods.

1. METHODOLOGY

Proposed Method:

The algorithm for proposed method is as follows:

Step 1: Find the smallest number (cost) of each column. Subtract this smallest number from every number in that column.

Step 2: Now add 1 to all element and we get at least one ones in each column. Then make assignment in terms of ones. If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step 3: Draw the minimum number of lines passing through all ones by using the following

procedure:

- (i) Mark ($\surd$) rows that do not have assignments.
- (ii) Mark ($\surd$) columns that have crossed ones in that marked rows.
- (iii) Mark ($\surd$) rows that have assignments in marked columns.
- (iv) Repeat (ii) and (iii) till no more rows or columns can be marked.
- (v) Draw straight lines through all unmarked rows and marked columns. If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step 4: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered numbers by this smallest number. Other numbers covered by lines remain unchanged. Then we get some new ones in row and column. Again make assignment in terms of ones.

Step 5: If we cannot get the optimal assignment in each row and column. Then repeat steps (3) and (4) successively till an optimum solution is obtained.

2. NUMERICAL PROBLEM

Problem 3.1: A department head has five subordinates, and five tasks have to be performed. Subordinates differ in efficiency and tasks differ in their intrinsic difficulty. Time each man would take to perform each task is given in the effectiveness matrix in Table 2. How the tasks should be allocated to each person so as to minimize the total man-hours?

Table 2: Effectiveness matrix

Subordinates →

Tasks↓	I	II	III	IV	V
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3

E	9	5	8	9	5
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3.1(a) Proposed Method: The following are the steps to solve the above assignment problem by using the proposed method for finding the optimal solution.

Step 1: Find the minimum element of each column and subtract it from each element in that column. Then the reduced matrix is shown in Table 3.

Table 3: Reduced matrix

Subordinates →

Tasks↓	I	II	III	IV	V
A	8	1	1	6	0
B	0	6	4	5	3
C	3	5	8	2	5
D	4	0	0	0	2
E	9	2	7	9	4

Step 2: Now add 1 to all the elements of the reduced matrix. It is shown in Table 4.

Table 4: Resulting matrix

Subordinates →

Tasks↓	I	II	III	IV	V
A	9	2	2	7	1
B	1	7	5	6	4
C	4	6	9	3	6
D	5	1	1	1	3
E	10	3	8	10	5

Step 3: Now make an initial assignment from the resulting matrix. It is shown in Table 5.

Table 5: Initial Assignment Matrix

Subordinates →

Tasks↓	I	II	III	IV	V
A	9	2	2	7	1
B	1	7	5	6	4
C	4	6	9	3	6
D	5	1	1	1	3
E	10	3	8	10	5

Here we have seen that 3rd row, 5th row, 2nd column and 4th column do not have any assignment. Thus the solution is not optimum, then we go to next step.

Step 4: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered number by this smallest number. Other numbers covered by lines remain unchanged. Then make assignment again. It is shown in Table 6.

Table 6: Final Assignment Matrix

Subordinates →

Tasks↓	I	II	III	IV	V
A	9	2	2	7	1
B	1	7	5	6	4
C	1.3	2	3	1	2
D	5	1 	1	1 	3
E	3.3	1	2.66	3.3	1.6

Now we have seen that all ones are either assigned or crossed out. That is, the total assigned one's is 5 which are equal to the number of rows or columns.

From original matrix, the minimum total man-hours are given in Table 7:

Table 7: Minimum Optimal Assignment

Optimal Assignment	A→V	B→I	C→IV	D→III	E→II	Total man- hour
Man-hour	1	0	2	1	5	9 hours

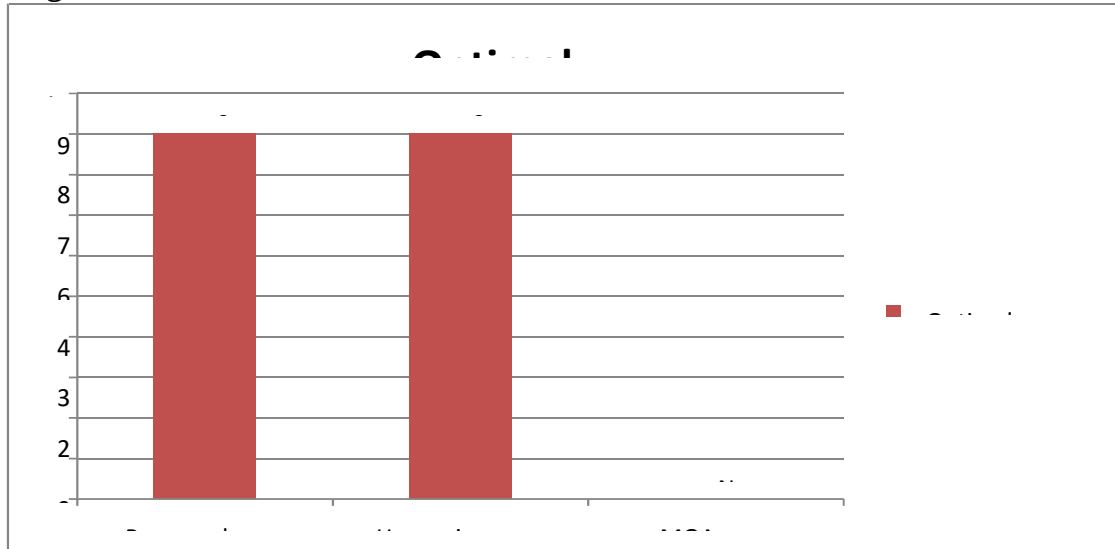
3.1(b) Comparison of the optimal solution of an assignment problem obtained by using proposed method with the well-known existing methods namely Hungarian Assignment method and MOA method. This is shown in Table 8.

Table 8: A comparison of an optimal solution of assignment problem by proposed method with Hungarian Assignment and MOA method

Methods	Value of the Objective Function (Assignment cost)	Optimal Solution
Proposed method	9	Optimal Assignment
Hungarian Assignment method	9	Optimal Assignment
MOA method	Not found	Not applicable

3.1(c) Graphical Representation of optimal solution for Assignment problem is shown in Figure 1.

Figure 1: Optimal Solution of Assignment problem by using proposed method, Hungarian method and MOA method



Problem 3.2: A company wishes to assign 5 jobs to 5 machines in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix (ij^{th} entry) is shown in Table 9. Find the optimum assignment schedule and minimum cost of making the assignment.

Table 9: Effectiveness matrix

Machines →

Jobs↓	I	II	III	IV	V
A	12	8	7	15	4
B	7	9	1	14	10
C	9	6	12	6	7
D	7	6	14	6	10
E	9	6	12	10	6

3.2(a) Proposed Method: The following are the steps to solve the above assignment problem by using the proposed method for finding the optimal solution.

Step 1: Find the minimum element of each column and subtract it from each element in that

column. Then the reduced matrix is shown in Table 10.

Table 10: Reduced matrix

Machines →

Jobs ↓	I	II	III	IV	V
A	5	2	6	9	0
B	0	3	0	8	6
C	2	0	11	0	3
D	0	0	13	0	6
E	2	0	11	4	2

Step 2: Now add 1 to all the elements of the reduced matrix. It is shown in Table 11.

Table 11: Resulting matrix

Machines →

Jobs ↓	I	II	III	IV	V
A	6	3	7	10	1
B	1	4	1	9	7
C	3	1	12	1	4
D	1	1	14	1	7
E	13	1	12	5	3

Step 3: Now make an initial assignment from the resulting matrix. It is shown in Table 12.

Table 12: Initial Assignment Matrix

Machines →

Jobs ↓	I	II	III	IV	V
A	6	3	7	10	1
B	1 ×	4	1	9	7
C	3	1 ×	12	1	4
D	1	1 ×	14	1 ×	7
E	13	1	12	5	3

Now we have seen that all ones are either assigned or crossed out. That is, the total assigned one's is 5 which are equal to the number of rows or columns. Hence an optimum assignment has been attained. From original matrix, the minimum assignment cost is given in Table 13:

Table 13: Minimum Optimal Assignment

Optimal Assignment	A→V	B→III	C→IV	D→I	E→II	Minimum Assignment cost (in Rs.)
Assignment Cost	4	1	6	7	6	24

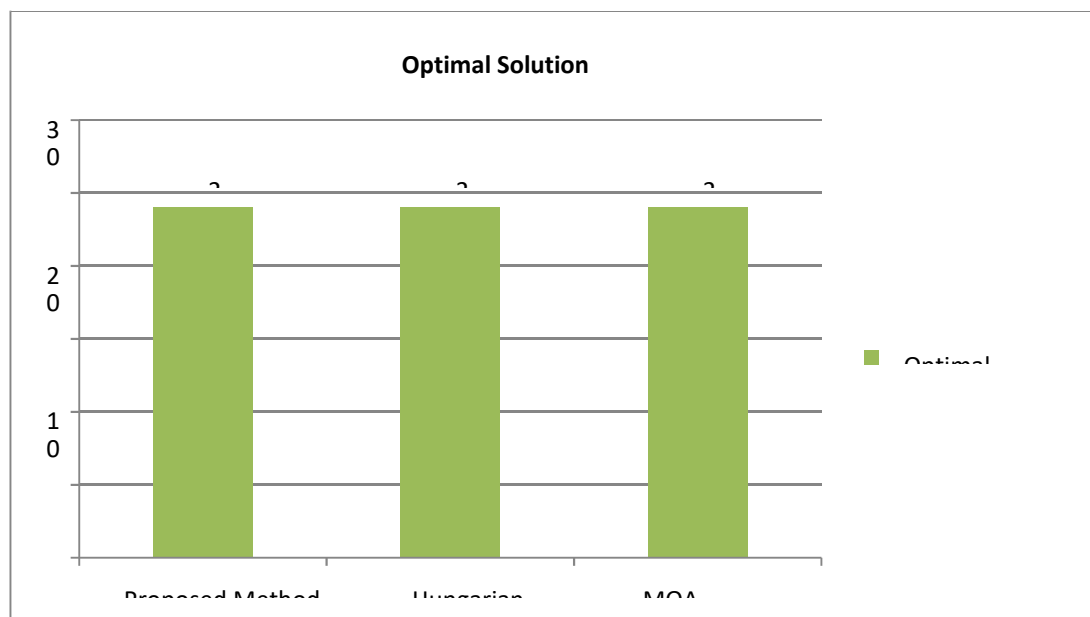
3.2(b) Comparison of the optimal solution of an assignment problem obtained by using proposed method with the well-known existing methods namely Hungarian Assignment method and MOA method. This is shown in Table 14.

Table 14: A comparison of an optimal solution of assignment problem by proposed method with Hungarian Assignment and MOA method

Methods	Value of the Objective Function (Assignment cost)	Optimal Solution
Proposed Method	24	Optimal Assignment and optimal Minimum cost
Hungarian Assignment Method	24	Optimal Assignment and optimal Minimum cost
MOA Method	24	Optimal Assignment and optimal Minimum cost

3.2(c) Graphical Representation of optimal solution of Assignment problem obtained by using proposed method, Hungarian method and MOA method is shown in Figure 2.

Figure 2: Optimal Solution of Assignment problem by using proposed method, Hungarian method and MOA method



3. RESULTS AND COMPARISON

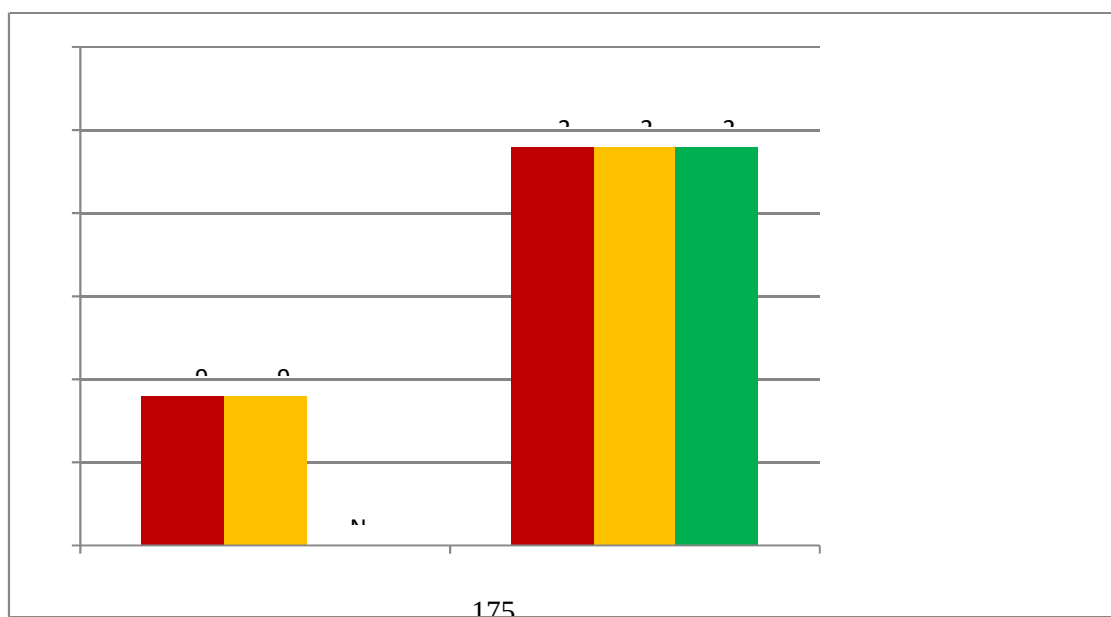
4.1 Comparison of the optimal solution of various types of assignment problems solved above by using the proposed method with the well-known existing methods namely Hungarian Assignment method and MOA method. This is shown in Table 15.

Table 15: A comparison of the methods– Proposed method, Hungarian Assignment method and MOA method

Methods	Problem 1	Problem 2	Optimal Solution
Proposed Method	9	24	Optimal Assignment and optimal Minimum cost
Hungarian Assignment Method	9	24	Optimal Assignment and optimal Minimum cost
MOA Method	-	24	Sometimes not applicable or Sometimes get optimal assignment

4.2 Graphical representation of the optimal solution of various types of assignment problems obtained by using new proposed method and the existing method namely Hungarian Assignment Method and MOA method. This is shown in Figure 3.

Figure 3: Optimal solution of Assignment problems using the proposed method, Hungarian method and MOA method



4. CONCLUSION

As we know we are having too many methods in our literature for finding an optimal assignment for assignment problems. Some of them were having some limitations which make those methods inefficient and is not suitable for finding an optimal assignment or minimum assignment cost for various cases of assignment problems. Keeping all these limitations or disadvantages in our mind we have proposed an alternate method for finding an optimal assignment of various types of assignment problems and this new approach overcomes all the limitations of the existing methods. Thus the proposed method is an efficient and reliable method for solving different types of Assignment problems. On observing the above comparison table, we conclude the efficiency of the proposed method and also get a global optimal assignment or minimum assignment cost which is similar to the optimal solution obtained by the well-known existing methods namely Hungarian Assignment method and Matrix Ones Assignment method. Thus we conclude that this paper attempts to propose an alternate approach which is easy to solve various real world situations of assignment problems as compared to existing ones.

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